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Article

Vague Translation in INK Algebra

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Abstract: In this paper we present the perceptions of Vague translation in INK algebra. We have introduced the notion of Vague set in INK algebra. The objective is to study features of Vague translation in ink algebra. The concept of Vague translation, Vague multiplication and Vague extension in INK algebra are discussed with the properties. Proved some examples to illustrate this concept and derived some theorem to get the structure of Vague translation, Vague multiplication and Vague extension in INK algebra and discussed the connection of Vague translation, Vague multiplication and Vague extension in ink algebra.

Keywords: Vague Translation, Vague Multiplication, Vague Extension, Vague INK Algebra, Vague subset

Introduction

W.L. Gau and D.J. Buehrer (1993) [1] introduced the concept of Vague sets in mathematics. Vague sets are an extension of fuzzy sets, where each value of the closed interval determines the boundary of truthiness. This is different from fuzzy sets, where each object is assigned a single value between 0 and 1. INK- algebras is generalized of BCI\BCK- algebras.

Two classes of abstract algebras BCK- algebras and BCI – algebras was introduced by Y. Imai (1966) [3] and K. Iseki (1980) [3]. It is known that the class of BCK- algebras and BCI- algebras is a proper subclass of the class of BCK- algebras. The notion of d- algebras which is another generalization of BCK- algebras was introduced by J. Neggers and H.S. Kim (1999) [9] . BH- algebras, which is a generalization of BCH\BCI\BCK- algebras was introduced by Y.B. Jun, E.H. Roh and H.S. Kim (1998) [4]. Huetal introduced a wide class of abstract namely BCH- algebras and also, we dispute of vague ink ideal in ink- algebras and studied some definitions, theorems and give some examples. INK- algebras is generalized of BCI\BCK- algebras. Notion of INK- algebras is generalized of BCI\BCK\BCH\Q\TM- algebras was introduced M. Kaviyarasum, Indhira Kandaiyan and V M. Chandrasekaran (2016)[7]. Fuzzy Translation on INK-Algebra was introduced by M. Kaviyarasu and K Indhira (2017)[8] on Fuzzy Translation on INK-Algebra.

Preliminaries

Definition: 2.1[6]

Let * be a binary operation on A with constant 0, Then a nonempty set (X ,*,0) is called a INK- algebra, if

1) $x * x = 0$,

2) $x * 0 = x, \forall x \in X$.

3) $0 * x = x$

4) $(y * x) * (y * z) = (x * z)$ for all $x, y, z \in X$.

Definition:2.2 [7]

An algebra (M, ∘, 0) is named a INK-algebra when it mollifies the subsequent orders for every x, z, $\alpha \in M$.

1) $((x \circ z) \circ (x \circ \alpha)) \circ (\alpha \circ z) = 0$
2) $((x \circ \alpha) \circ (z \circ \alpha)) \circ (x \circ z) = 0$
3) $x \circ 0 = x$
4) $x \circ z = 0$ and $z \circ x = 0$ imply $x = z$.

Definition: 2.3 [6]

A non-empty subset S of a INK-algebra (X, *, 0) is said to be a INK-sub algebra of X, if $a * b \in S$, whenever a , b $\in X$.

Definition: 2.4[7]

Consider a subset A of a INK- algebra M is named an INK-ideal of M if,

1. $0 \in A$,
2. $(\alpha \circ x) \circ (\alpha \circ z) \in$ A and $\tau \in A$ imply $x \in A$. $x, z, \alpha \in M$.

Definition: 2.5[7]

A FS V_A in a INK-algebra X is named a F-INK- ideal of X, if,

1. $V_A(0) \geq V_A(x)$
2. $V_A(z) \geq \min\{V_A(z \circ x) \circ (z \circ \alpha), V_A(z)\}$, $x, z, \alpha \in X$.

Definition: 2.6[7]

A FS V_A in a INK-algebra X is named a F-INK-sub algebra (F-INK) of X, if,

$V_A(x \circ z) \geq \min\{V_A(x), V_A(z)\}$, $x, z \in X$.

Vague Translation and Vague Multiplication in INK-Algebra

Definition: 3.1

Consider $V_A = [t_A, 1\text{-}f_A]$ represents a vague subset of X and Y ∈ [0, T]. A mapping $V_A(Y)^T$ from X→ [0, 1] is said to be Vague-Y -translation (V-Y) of V_A, when it completes $(V_A)_Y^T$ (x) = V_A(x)+Y, for all x ∈ X. $(t_A)_Y^T$ (x) = t_A(x)+Y: $(1 - f_A)_Y^T$ (x) = 1-f_A(x)+Y

Definition: 3.2

Let $V_A = [t_A, 1\text{-}f_A]$ indicates a vague subset of X and $Y \in [0,1]$. A mapping $(V_A)_Y^X$: $X \to [0, 1]$ is said to be a V-Y -multiplication of V_A when it completes $(V_A)_Y^X$ (x) = $Y\, V_A(x)$, x in X.

Example: 3.3 Let X= {0, 1, 2} represent the collection with the subsequent table.

o	a	b	c
a	a	a	c
b	b	a	b

c	c	c	a

a=0; b=1; c=2

Define a vague set V_A = [t_A, 1-f_A] in X as follows:

X	0	1	2
V_A(x)	[0.40,0.50]	[0.30,0.20]	[0.70,0.80]

Then V_A is a vague X and T. if we take Y = 0.12, $(V_A)^T$ = [$(t_A)^T$, $(1\text{-}f_A)^T$]

X	0	1	2
$(V_A)_Y^T$	[0.048,0.06]	[0.036,0.024]	[0.084,0.096]

Then $(V_A)_Y^T$ is also a vague multiplication of X.

Theorem: 3.4 If V_A = [t_A, 1-f_A] of X is a VINKSA(Vague-INK-subalgebra) and Y, ∈ [0, T], subsequently the V-Y -translation $(V_A)_Y^T$ (x) of V_A is also a VINKSA of X.

Proof. Let x, $z \in X$ and $Y \in$ [0, T].

Then $t_A(x \circ z) \geq \min\{t_A(a), t_A(b)\}$

Now $(t_A)_Y^T(x \circ z) = t_A(x \circ z) + Y$

$\geq \min\{t_A(x), t_A(z)\} + Y$

$= \min\{t_A(x) + Y, t_A(z) + Y\}$

$= \min\{(t_A)_Y^T(x), (t_A)_Y^T(z)\}$.

Then $1\text{-}f_A(x \circ z) \geq \min\{1\text{-}f_A(a), 1\text{-}f_A(b)\}$

Now $(1 - f_A)_Y^T(x \circ z) = 1\text{-}f_A(x \circ z) + Y$

$\geq \min\{1\text{-}f_A(x), 1\text{-}f_A(z)\} + Y$

$= \min\{1\text{-}f_A(x) + Y, 1\text{-}f_A(z) + Y\}$

$= \min\{(1 - f_A)_Y^T(x), (1 - f_A)_Y^T(z)\}$.

Theorem: 3.5 Consider V_A = [t_A, 1-f_A] indicate a Vague-subset of X such that the V-Y- translation $(V_A)_Y^T$ of V_A is a VINKSA of X for certain $Y \in$ [0, T], subsequently V_A indicates a VINKSA of X.

Proof. Presume that $(t_A)_Y^T$ is a VSA of X for certain Y, ∈ [0, T].

Consider x, $z \in$ X. then we got

$t_A(x \circ z) + Y = (t_A)_Y^T(x \circ z)$

$\geq \min\{(t_A)_Y^T(x), (t_A)_Y^T(z)\}$

$= \min\{t_A(x) + Y, t_A(z) + Y\}$

$= \min\{t_A(x), t_A(z)\} + Y$

$t_A(x \circ z) \geq \min\{t_A(x), t_A(z)\}$ for all $x, z \in X$.

Presume that $(1 - f_A)_Y^T$ is a VSA of X for certain $Y \in$ [0, T].

Consider x, $z \in$ X. then we got

$1\text{-}f_A(x \circ z) + Y = (1 - f_A)_Y^T(x \circ z)$

≥ min {$(1-f_A)_Y^T$ (x), $(1-f_A)_Y^T$ (z)}

= min {1-f_A (x) +Y, 1-f_A (z) +Y}

= min {1-f_A (x), 1-f_A (z)} + *Y*

1-f_A (*x*◦ *z*) ≥ min {1-f_A (x), 1-f_A (z)} for all x, *z*∈ *X*.

Hence V_A = [t_A, 1-f_A] is VIMKSA of *X*.

Theorem: 3.6 In case if VINKSA V_A = [t_A, 1-f_A] of *X* and *Y* ∈ [0, *T*], if the V-*Y*-multiplication $(V_A)_Y^X$ (x) of V_A = [t_A, 1-f_A] is a VINKSA of *X*.

Proof. Let *x, z* ∈ *X* and *Y* ∈ [0, Ţ].

t_A (*x* ∘ *z*) ≥ min {t_A(x), t_A(z)}.

$(t_A)_Y^X$ (x ∘ z) = Y. t_A (x ∘ z)

≥ *Y* min {t_A(x), t_A(z)}

= min {Y. t_A(x).Y.t_A(z)}

= min { $(t_A)_Y^X$ (x), $(t_A)_Y^X$ (z)}

$(t_A)_Y^X$ (x ∘ z) ≥ min {$(t_A)_Y^X$ (x), $(t_A)_Y^X$ (z)}

1-f_A (*x* ∘ *z*) ≥ min {1-f_A (x), 1-f_A (z)}.

$(1-f_A)_Y^X$ (x ∘ z) = Y. 1-f_A (x ∘ z)

≥ *Y* min {1-f_A (x), 1-f_A (z)}

= min {Y. 1-f_A (x), Y. 1-f_A (z)}

= min { $(1-f_A)_Y^X$ (x), $(1-f_A)_Y^X$ (z)}

$(1-f_A)_Y^X$ (x ∘ z) ≥ min { $(1-f_A)_Y^X$ (x), $(1-f_A)_Y^X$ (z)}.

Hence $(V_A)_Y^T$ is VINKSA of *X*.

Theorem: 3.7 For whichever Vague-subset V_A = [t_A, 1-f_A] of *X* and *Y*, ∈ [0, Ţ], if the V-*Y* -multiplication $(V_A)_Y^X$ (x) of $(V_A)_Y^X$ (z) is a VINKSA of *X*, then so is V_A.

Proof. Assume that $(t_A)_Y^X$ (x) of t_A is a VINKSA of *X* for some *Y* ∈ [0, *Ţ*].

Let *x, z* ∈ *X*. We have

Y. t_A (x ∘ z) = $(t_A)_Y^X$ (x ∘ z)

≥ min { $(t_A)_Y^X$ (x), $(t_A)_Y^X$ (z)}

= min {Y. t_A(x),Y.t_A(z)}

= *Y*. min {t_A(x), t_A(z)}

t_A (*x* ∘ *z*) ≥ min {t_A(x), t_A(z)}.

Y. 1-f_A (x ∘ z) = $(1-f_A)_Y^X$ (x ∘ z)

≥ min { $(1-f_A)_Y^X$ (x), $(1-f_A)_Y^X$ (z)}

= min {Y. 1-f_A (x)Y. 1-f_A (z}

= *Y*. min {1-f_A (x), 1-f_A (z)}

1-f_A ($x \circ z$) ≥ min {1-f_A (x), 1-f_A (z)}.

4. Vague Extension on INK-Algebra

Definition: 4.1 Consider V_{A1} and V_{A2} indicates vague subsets of *X*. If V_{A1} (*x*) ≤ V_{A2} (*x*) for all *x* in *X*, and subsequently it is considered that V_{A2} is a vague extension (V*E*) of V_{A1}.

$t_{A1}(x) \leq t_{A2}(x)$, 1 - $f_{A1}(x) \leq 1 - f_{A2}(x)$

Definition: 4.2 Consider $V_{A1}$and V_{A2} indicate vague subsets of *X*. Subsequently, V_{A2} is known as a vague Š-extension (VŠ) of V_{A1} when the subsequent statements are applicable:

1) V_{A2} indicates a fuzzy extension of V_{A1}.
2) When V_{A1} represents a VINKSA of *X*, in that case V_{A2} is a VINKSA of *X*.

Through the characterization of vague-*Y*-translation, it is absolutely clear that $(V_A)_Y^T$ ≥ V_A *(x), x* ∈ *X*.

For this reason, the subsequent Theorem is formulated.

Theorem: 4.3 Consider V_A = [t_A, 1-f_A] represents a Vague-subset of *X* and *Y*∈ [0, *T*]. Subsequently, the V-*Y*-translation $(V_A)_Y^T$ of V_A is a VINKSA of *X* if and only if *X Y,* (V_A; *t*) represents a VINKSA of *X* for the entire *t* ∈ Im (V_A) with *t* ≥ *Y*.

Proof. Indispensable part is obvious. In order to demonstrate the adequacy,

Consider that there presents *a*, b ∈ *X* in order that

$(t_A)_Y^T$ ($a \circ b$) < α ≤ min { $(t_A)_Y^T$ (*a*), $(t_A)_Y^T$ (*b*)}.

t_A (a) ≥ $\alpha - Y$ and t_A (b) ≥$\alpha - Y$ but t_A (a ∗ b) <$\alpha - Y$.

This demonstrates that *a, b* ∈ *X* (t_A; α) & a∘ b ∈ (*X*, t_A; α).

It is a negation, and so

$(t_A)_Y^T$ ($x \circ z$) ≥ min { $(t_A)_Y^T$ (*x*), $(t_A)_Y^T$ (*z*)} for all *x, z* ∈ *M*.

$(1 - f_A)_Y^T$ ($a \circ b$) < α ≤ min { $(1 - f_A)_Y^T$ (*a*), $(1 - f_A)_Y^T$ (*b*)}.

1-f_A (a) ≥ $\alpha - Y$ and 1-f_A (b) ≥ $\alpha - Y$ but 1-f_A (a ∗ b) <$\alpha - Y$

This demonstrates that *a, b* ∈ *X* (1-f_A; α) & a∘ b ∈ (X_Y, 1-f_A; α).

It is a negation, and so

$(1 - f_A)_Y^T$ ($x \circ z$) ≥ min { $(1 - f_A)_Y^T$ (*x*), $(1 - f_A)_Y^T$ (*z*)} for all *x, z* ∈ *X*

Hence $(V_A)_Y^T$ is a VINKSA of *X*.

Theorem: 4.4 Let V_A = [t_A, 1-f_A] be a VINKSA of *X*. In addition, let *Y*, α∈ [0, T] When *Y* ≥ α, at that moment the V-*Y*-translation $(V_A)_\alpha^T$ T of V_A is a VŠ-extension of the F-α-translation $(V_A)_\alpha^T$ of V_A.

For each VINKSA V_A of $\tilde{N}$ & α∈ [0, T], the V-α-translation $(V_A)_\alpha^T$ of V_A is a VINKSA of *X*. When ν indicates a VŠ-extension of $(V_A)_\alpha^T$ at that moment there exists *Y*∈ [0, *T*] in order that

Y ≥ α& ν(*x*) ≥ $(V_A)_\alpha^T$ (*x*), for the entire *x*∈ *X*.

Theorem: 4.5 Consider V_A = [t_A, 1-f_A] represents a VINKSA of *X* and $\alpha \in$ [0, T]. For each and every VŠ-extension ν of the V-α-translation $(V_A)_\alpha^T$ of V_A, there presents *Y*∈ [0, *T*] in order that $Y \geq \alpha$ & ν indicates a VŠ-extension of the V-*Y*-translation $(V_A)_Y^T$ of V_A.

Example: 4.6. Let a INK-algebra *X*= {0,1,2,3,4} with the Cayley table

o	a	b	c	d	e
a	a	a	a	a	a
b	b	a	a	b	b
c	c	c	a	c	c
d	d	d	d	a	d
e	e	e	e	e	a

a=0; b=1; c=2; d=3; e=4

Define a V-subset V_A of X by

X	a	b	c	d	e
V_A	[0.70,0.40]	[0.20,0.50]	[0.10,0.90]	[0.60,0.30]	[0.80,0.40]

If we take $\alpha = 0.2$, then the V- α-translation $(V_A)_Y^T$ of V_A is given by

X	a	b	C	d	e
$(V_A)_Y^T$	[0.90,0.60]	[0.40,0.70]	[0.30,1.10]	[0.80,0.50]	[1.0,0.60]

Definition: 4.7 Consider V_A = [t_A, 1-f_A] represents a Vague-subset of *X* and β∈ [0, 1]. A vague-β -multiplication $V_A(\beta)$ of V_A, indicated by $V_A(\beta)$ m, is characterized to be a plot $(V_A)_\beta^m : X \to$ [0, 1], *x* V_A *(x)* · β. In case of any vague subset V_A of *X*, a vague 0-multiplication $(V_A)_o^m$ of V_A is evidently a VSA(Vague-subalgebra) of *X*.

Theorem: 4.8 When V_A represents a VSA of *X*, subsequently the $(V_A)_\beta^m$ of V_A is a VINKSA of *X* for all $\beta \in$ [0, 1].

Theorem: 4.9 For each Vague-subset V_A of *X*, the subsequent are counterpart:

(i) V_A indicates a VINKSA of *X*.

(ii) (∀$\beta \in$ (0, 1]) ($(V_A)_\beta^m$ represents a VINKSA of *X*).

Proof. Indispensable part goes after from Theorem 4.8. Consider $\beta \in$ (0, 1] be in order that $(V_A)_\beta^m$ is a VINKSA of *M*. Subsequently

$t_A (x \circ z) \cdot \beta = (t_A)_\beta^m (x \circ z)$

$\geq \min \{(t_A)_\beta^m (x), (t_A)_\beta^m (z)\}$

$= \min \{t_A (x) \cdot \beta, t_A (z) \cdot \}$

$= \min \{t_A (x), t_A (z)\} \cdot \beta$

$t_A (x \circ z) = \min \{t_A (x), t_A (z)\}$, for the entire *x*, *z* ∈ X in view of the fact that $\beta \neq 0$.

1-f_A (x∘ z) · β = $(1-f_A)^m_\beta$ (x∘ z)

≥ min {$(1-f_A)^m_\beta$ (x), $(1-f_A)^m_\beta$ (z)}

= min {1-f_A (x) ·β, 1-f_A (z) · }

= min {1-f_A (x), 1-f_A (z)} · β

1-f_A (x∘ z) = min {1-f_A (x), 1-f_A (z)}, for the entire x, z ∈ X in view of the fact that β ≠0. For this reason V_A= [t_A, 1-f_A] indicates a VINKSA of X

Theorem: 4.10 Consider V_A= [t_A,1-f_A] represents a Vague-subset of $\tilde{N}$, Y∈ [0, Ʈ] & β ∈ (0, 1]. Subsequently, the entire V-Y-translation $(V_A)^T_Y$ of V_A indicates a VŠ-extension of the V-γ -multiplication $(V_A)^X_\beta$ of V_A.

Proof. For each x∈ X, it is known that $(t_A)^T_Y$ (x) = t_A (x) + Y ≥ t_A (x) ≥ t_A (x) · β = $(t_A)^X_\beta$ (x)

And so, $(t_A)^T_Y$ indicates a VE-extension of $(t_A)^X_\beta$ .Take that $(t_A)^X_\beta$ represents a VINKSA of X.

Subsequently, V_A indicates a VINKSA of X in Theorem 4.5. It pursues from Theorem 3.4 that $(t_A)^T_Y$ indicates a VINKSA of X for every Y∈ [0, T].

Therefore, the entire V-Y-translation $(t_A)^T_Y$ indicates a VŠ-extension of the V- β -multiplication $(t_A)^X_\beta$.

For each x∈ X, it is known that $(1-f_A)^T_Y$ (x) = 1-f_A (x) + Y ≥ 1-f_A (x) ≥ 1-f_A (x) · β = $(1-f_A)^X_\beta$ (x)

and so, $(1-f_A)^T_Y$ indicates a VE-extension of 1-f_A $(\beta)^x$. Take that $(1-f_A)^X_\beta$ represents a VINKSA of X.

Subsequently, V_A indicates a VINKSA of X in Theorem 4.5. It pursues from Theorem 3.4 that $(1-f_A)^T_Y$ indicates a VINKSA of X for every Y∈ [0, T].

Therefore, the entire V-Y-translation $(1-f_A)^T_Y$ indicates a VŠ-extension of the V-β -multiplication$(1-f_A)^X_\beta$.

Conclusion

In this chapter, Vague translation and Vague extension of INK algebra have been introduced in the INK algebra. Vague translation in INK algebra is deal with the concept that isn't exactly clear or precise. INK algebra had been observed that it satisfies the environments stated of BCI\BCK-algebras. The idea of Vague are use to work with the imprecise information and make math more adaptable to real world situation. The concepts of Vague translation, Vague multiplication and Vague extension of INK algebra are discussed in this chapter and some examples solved with this concept. The Vague translation are deal where it is not strictly true or false and transform some concept of mathematical ideas while keep the flexibility.

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Does this article screen for similarity? Yes

Conflict of Interest: The Authors have no conflicts of interest to declare that they are relevant to the content of this article.

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